

UNCLASSIFIED

---

AD 294 422

*Reproduced  
by the*

ARMED SERVICES TECHNICAL INFORMATION AGENCY  
ARLINGTON HALL STATION  
ARLINGTON 12, VIRGINIA



---

UNCLASSIFIED

NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.

Contract Number AF 04(647)-243  
Exhibit D Paragraph 3.43

CONVERSION OF STRESS RELAXATION MODULUS  
TO DYNAMIC MODULUS  
FOR A VISCOELASTIC MATERIAL

MT0-270-2

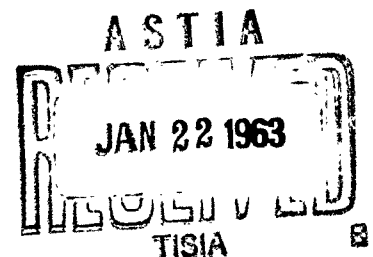
November 1962

Prepared by

HERCULES POWDER COMPANY  
CHEMICAL PROPULSION DIVISION  
Bacchus Works  
Magna, Utah

Prepared for

HEADQUARTERS  
SPACE SYSTEMS DIVISION  
UNITED STATES AIR FORCE  
Los Angeles, California



Report No. MTO-270-2

Copy No. \_\_\_\_\_

Date November 1962

CONVERSION OF STRESS RELAXATION MODULUS  
TO DYNAMIC MODULUS  
FOR A VISCOELASTIC MATERIAL

Approved by

*T. L. Vose*

\_\_\_\_\_  
T. L. Vose  
Superintendent  
Structural Dynamics Group

Prepared by

*J. M. Anderson*

\_\_\_\_\_  
J. M. Anderson  
Development Engineer  
Structural Dynamics Group

## FOREWORD

This document describes techniques used in determining material properties of the Stage III, XM-57 Minuteman Motor. It was prepared by the Structural Dynamics Group, Research & Development Department, Bacchus Works, Chemical Propulsion Division, Explosives Department, Hercules Powder Company.

Authority for the preparation of this report was obtained from contract AF 04(647)-243, Exhibit D, paragraph 3.43.

Published by

The Publications Group  
Graphic Services Department  
HERCULES POWDER COMPANY  
Bacchus Works  
Magna, Utah

# ABSTRACT

The relationship between the stress relaxation modulus,  $E(t)$ , and the dynamic modulus,  $E^*(i\omega) = E'(\omega) + i E''(\omega)$ , can be expressed in the form

$$E'(\omega) = \omega \int_0^{\infty} E(t) \sin \omega t \, dt$$

and

$$E''(\omega) = \omega \int_0^{\infty} E(t) \cos \omega t \, dt$$

The evaluation of these integrals must be a numerical process since  $E(t)$  as obtained from test data is ordinarily given in tabular form. The basic numerical approach presented in this document is to fit  $E(t)$  over each time definition interval with a function of the type

$$E(t) \left| \begin{array}{l} t_{n+1} \\ t_r \end{array} \right. = A_n e^{-b_n t}$$

The integral defining  $E'(\omega)$  can then be expressed as sums of integrals of the type

$$\int_{t_n}^{t_{n+1}} A_n e^{-b_n t} \sin \omega t \, dt$$

This integral and the companion integral for  $E''(\omega)$  can be evaluated analytically, resulting in a series approximation for  $E^*(i\omega)$  containing no integrals.

The numerical procedure is carried out using a digital computer. An accuracy investigation is performed using the tensile stress relaxation data for polyisobutylene. The regime in which  $E(t)$  must be known in order to accurately compute  $E^*(i\omega)$  is determined for a frequency spectrum.

# TABLE OF CONTENTS

| <u>Section</u>  |   | <u>Page</u> |
|-----------------|---|-------------|
|                 | FOREWORD . . . . .                                      | iii         |
|                 | ABSTRACT . . . . .                                      | iv          |
|                 | LIST OF FIGURES . . . . .                               | vi          |
|                 | LIST OF TABLES . . . . .                                | vi          |
| I               | INTRODUCTION . . . . .                                  | 1           |
| II              | METHOD OF SOLUTION . . . . .                            | 2           |
| III             | DISCUSSION OF RESULTS . . . . .                         | 9           |
|                 | LIST OF REFERENCES . . . . .                            | 15          |
|                 | LIST OF SYMBOLS . . . . .                               | 16          |
| <u>Appendix</u> |   | <u>Page</u> |
| A               | 1620 COMPUTER PROGRAM . . . . .                         | A-1         |
|                 | 1. Operating Instructions and Input Data Format . . . . | A-1         |
|                 | 2. Program Listing . . . . .                            | A-2         |

# LIST OF FIGURES

| <u>Number</u> | <u>Title</u>  | <u>Page</u> |
|---------------|---|-------------|
| 1             | Exponential Fit of Polyisobutylene Stress Relaxation<br>Data for the Time Interval Between 10 and 100 Hours . . . | 4           |
| 2             | $E^*(i\omega)$ for Polyisobutylene . . . . .  | 10          |
| 3             | Time Regime in which $E(t)$ Must be Known in Order to<br>Compute $E^*(i\omega)$ for Polyisobutylene . . . . .     | 11          |

# LIST OF TABLES

| <u>Number</u> | <u>Title</u>   | <u>Page</u> |
|---------------|--|-------------|
| I             | Stress Relaxation Data for Polysiobutylene . . . . . | 12          |
| II            | Interval Contributions for $E^*(i\omega)$ . . . . .  | 13          |



## SECTION I

### INTRODUCTION

The stress relaxation modulus,  $E(t)$ , for a linear viscoelastic material can be converted to the dynamic modulus,  $E^*(i\omega) = E'(\omega) + iE''(\omega)$ , by evaluating the following integrals:<sup>1</sup>

$$E'(\omega) = \omega \int_0^{\infty} E(t) \sin \omega t \, dt$$
$$E''(\omega) = \int_0^{\infty} E(t) \cos \omega t \, dt$$

It is not possible to obtain  $E(t)$  from tests in the time range specified by either the lower or upper limit of these integrals. Therefore, what must the time range be for  $E(t)$  in order to obtain a good approximation for  $E^*(i\omega)$ ? The data from stress relaxation tests is ordinarily obtained in tabular form. A numerical procedure must be used to evaluate the integrals for  $E'(\omega)$  and  $E''(\omega)$ .

---

<sup>1</sup>See List of References.

## SECTION II

### METHOD OF SOLUTION

Given the time dependent relaxation modulus,  $E(t)$ , for a linear viscoelastic material, it is desired to compute the frequency dependent dynamic modulus,  $E^*(i\omega) = E'(\omega) + i E''(\omega)$ . The relationship between  $E^*(i\omega)$  and  $E(t)$  can be expressed in several forms. Commonly used forms for this relationship are:<sup>1,2</sup>

$$E'(\omega) = \omega \int_0^{\infty} E(t) \sin \omega t \, dt \quad (1)$$

$$E''(\omega) = \omega \int_0^{\infty} E(t) \cos \omega t \, dt \quad (2)$$

and

$$E'(\omega) = E_c + \omega \int_0^{\infty} \psi(t) \sin \omega t \, dt \quad (3)$$

$$E''(\omega) = \omega \int_0^{\infty} \psi(t) \cos \omega t \, dt \quad (4)$$

where:  $\psi(t) = E(t) - E_c$

$E_c$  = constant value approached by  $E(t)$

It can be shown that equations (1) and (3) are equivalent, as are equations (2) and (4). This demonstration is based upon the fact that

$$\int_0^{\infty} \sin x \, dx = 1 \quad \text{and} \quad \int_0^{\infty} \cos x \, dx = 0$$

The integral in equation (1) can be written as

$$\int_0^{\infty} E(t) \sin \omega t \, dt = \int_0^{\frac{2m\pi}{\omega}} E(t) \sin \omega t \, dt + \int_{\frac{2m\pi}{\omega}}^{\infty} E_c \sin \omega t \, dt$$

where  $m$  is an integer chosen such that  $E(t) = E_c$  for all  $t > \frac{2m\pi}{\omega}$

---

<sup>1,2</sup> See List of References

Since

$$\omega \int_{\frac{2m\pi}{\omega}}^{\infty} E_c \sin \omega t \, dt = E_c \quad \text{and} \quad \omega \int_{\frac{2m\pi}{\omega}}^{\frac{2m\pi}{\omega} + \frac{\pi}{2\omega}} E_c \sin \omega t \, dt = E_c$$

equation (1) can be written as

$$E'(\omega) = \omega \int_0^{\frac{2m\pi}{\omega} + \frac{\pi}{2\omega}} E(t) \sin \omega t \, dt. \quad (5)$$

A similar treatment of equation (2) yields

$$E''(\omega) = \omega \int_0^{\frac{2m\pi}{\omega}} E(t) \cos \omega t \, dt. \quad (6)$$

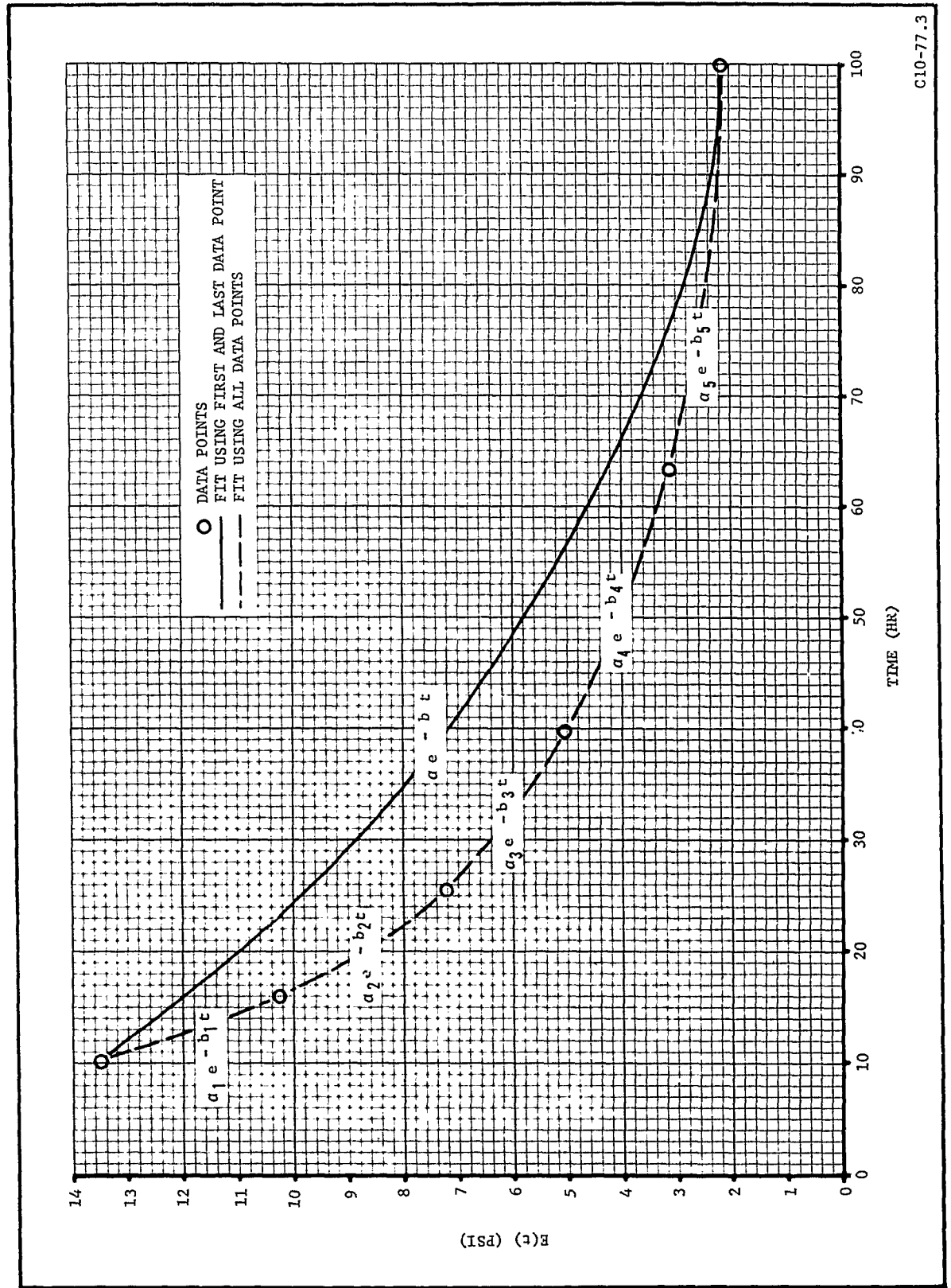
Because  $E(t)$ , as obtained from tests, is in tabular form, a numerical method must be used to evaluate the integrals for  $E^*(i\omega)$ . A procedure for curve fitting will be necessary in any method chosen. Two distinct types of curve fitting methods are available. One type consists of determining a single function which is good for the full range of the independent variable. This is the type of fit accomplished when a multiparameter Maxwell model is assumed for the material.<sup>3</sup> The other type of curve fit method consists of selecting discrete functions to be used in different ranges of the independent variable. The latter method was chosen for further study because of its simplicity and accuracy potential.

Examination of several sets of tabular data for  $E(t)$  has indicated that  $E(t)$  can be fit over time intervals with functions of the type

$$E(t) \Big|_{t_n}^{t_{n+1}} = a_n e^{-b_n t} \quad (\text{as shown in Figure 1})$$

---

<sup>3</sup> See List of References



C10-77.3

Figure 1. Exponential Fit of Polyisobutylene Stress Relaxation Data for the Time Interval Between 10 and 100 Hours

Substitution of this expression into equations (5) and (6) yields

$$E'(\omega) = \omega \sum_{n=1}^{n_1} \int_{t_n}^{t_{n+1}} a_n e^{-b_n t} \sin \omega t dt \quad (7)$$

$$E''(\omega) = \omega \sum_{n=1}^{n_2} \int_{t_n}^{t_{n+1}} a_n e^{-b_n t} \cos \omega t dt \quad (8)$$

where  $n_1$  is chosen such that

$$t_{n_1} = \frac{2m\pi}{\omega} + \frac{\pi}{2\omega}$$

and  $n_2$  is chosen such that

$$t_{n_2} = \frac{2m\pi}{\omega}$$

For the time interval between  $t_n$  and  $t_{n+1}$ , these integrals can be evaluated analytically, the result being

$$E'(\omega) = \omega \sum_{n=1}^{n_1} \frac{a_n}{b_n^2 + \omega^2} \left[ e^{-b_n t} (b_n \sin \omega t + \omega \cos \omega t) \right]_{t_{n+1}}^{t_n} \quad (9)$$

and

$$E''(\omega) = \omega \sum_{n=1}^{n_2} \frac{a_n}{b_n^2 + \omega^2} \left[ e^{-b_n t} (b_n \cos \omega t - \omega \sin \omega t) \right]_{t_{n+1}}^{t_n} \quad (10)$$

where:  $\omega$  = given circular frequency

$$b_n = \ln \left[ \frac{E(t_n)}{E(t_{n+1})} \right]$$


---


$$a_n = E(t_n) e^{b_n t_n}$$

Two considerations should be kept in mind when the time increments,  $t_n$ , in equations (9) and (10) are chosen. The first consideration is that for small values of  $t$ ,  $E(t)$  changes rapidly with time. The  $t_n$  should be chosen in this time regime where the test data points occur. The second consideration concerns convergence.

A method of determining the convergence of the integrals in equations (5) and (6) should be provided. A convergence criterion is easily obtained if  $t_{n+1}$  is chosen such that  $\sin \omega t_{n+1} = \sin \omega t_n$  and  $\cos \omega t_{n+1} = \cos \omega t_n$ . If  $t_n$  is large compared with  $2\pi/\omega$ , then the data points thereafter can be assumed to occur at multiples of  $2\pi/\omega$  with little error.

Assume  $t_{n+1} = t_n + 2 \frac{p\pi}{\omega}$  in equation (9), where  $p$  is a given positive integer. Further, assume that  $a_n$ ,  $b_n$ , and  $\omega$  are given for the interval  $t_n$  to  $t_{n+1}$ . Then the interval contribution,  $\delta_n$ , is equal to

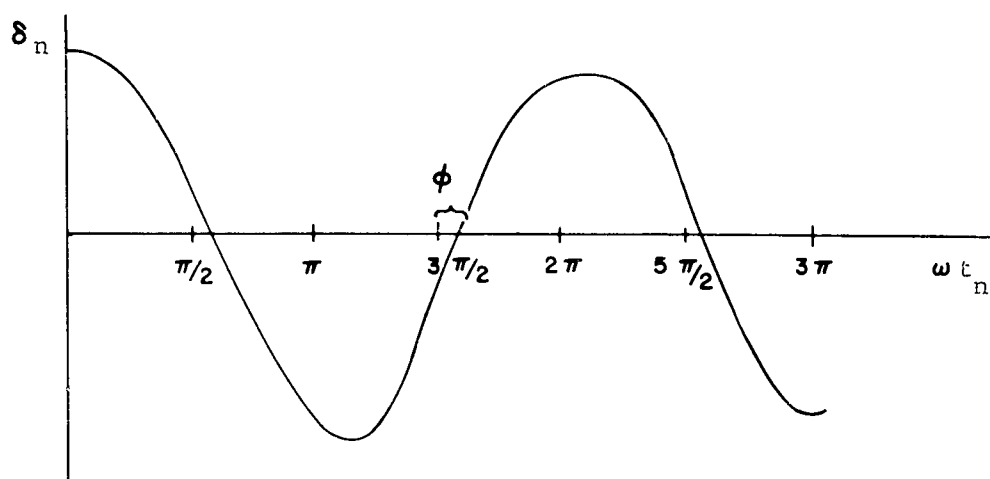
$$\delta_n \left| \begin{array}{c} t_n + \frac{2p\pi}{\omega} \\ t_n \end{array} \right. = \frac{a_n}{b_n^2 + \omega^2} \left[ e^{-b_n t} (b_n \sin \omega t + \omega \cos \omega t) \right]_{t_n + \frac{2p\pi}{\omega}}^{t_n}$$

This equation can be expressed in the form

$$\delta_n \left| \begin{array}{c} t_n + \frac{2p\pi}{\omega} \\ t_n \end{array} \right. = \frac{a_n}{b_n^2 + \omega^2} (1 - e^{-2p\pi \frac{b_n}{\omega}}) \left\{ e^{-b_n t_n} \sin (\omega t_n + \phi) \right\} \quad (11)$$

where  $\phi = \tan^{-1} \left( \frac{\omega}{b_n} \right)$

A plot of  $\delta_n$  as function of  $t_n$  will appear as



The interval contribution is a maximum when  $t_n = q \frac{\pi}{2\omega} - \frac{\phi}{\omega}$ , when  $q$  is an even integer, and the contribution is zero when  $q$  is an odd integer.

As  $E(t)$  approaches a constant,  $b_n$  and  $\phi$  approach zero. Thus  $\delta_n$  is small when  $t_n$  is an odd multiple of  $\pi/2\omega$ . This fact, coupled with the fact that  $t_{n1}$  must be an odd multiple of  $\frac{\pi}{2\omega}$ , has the implication that the  $t_n$  in equation (9) should be chosen as odd multiples of  $\pi/2\omega$  when  $t_n$  is sufficiently large to permit the approximation. With the  $t_n$  thus chosen, equation (11) reduces to

$$\delta_n \left|_{t_n}^{t_{n+1}} = \frac{b_n}{\omega} \frac{1}{1 + \left(\frac{b_n}{\omega}\right)^2} \left[ E(t_n) - E(t_{n+1}) \right] \quad (12)$$

A similar discussion and treatment of  $E''(\omega)$  implies that  $t_n$  in equation (10) should be chosen as even multiples of  $\pi/2\omega$  when  $t_n$  is sufficiently large to permit the approximation. With these assumptions concerning  $t_n$ , the interval contribution formula for  $E''(\omega)$  is identical with equation (12).

A computer program was written for the IBM 1620 digital computer. (See Appendix A.) This program evaluates equations (9), (10), and (12). The time intervals,  $t_n$  and  $t_{n+1}$ , in equations (9) and (10) are chosen as the intervals at which the data points are given. When a  $t_n$  is found such

that  $t_n > K \left( \frac{2\pi}{\omega} \right)$ , where  $K$  is an integer given as input, the program assumes that  $E(t_n)$  is equal to  $E(t_p)$  where  $\omega t_p = \left( \frac{\pi}{2} + \text{nearest integral multiple of } 2\pi \right)$  in equation (9) and  $\omega t_p = \text{nearest integral multiple of } 2\pi$  in equation (10). Thereafter, the program uses equation 12 to compute  $E^*(i\omega)$ .



### SECTION III

#### DISCUSSION OF RESULTS

To analyze the accuracy of the method, the stress relaxation data for polyisobutylene<sup>4</sup> (see table I) was run using this computer program. The stress relaxation data extends over 16 decades in time, and is acknowledged to be the best data available at the present time. The computer results for  $E^* (i\omega)$  are shown in figure 2. The data for the dynamic shear modulus,  $G^* (i\omega)$ , for polyisobutylene<sup>5</sup> was converted to  $E^* (i\omega)$  using the formula  $E^* (i\omega) = 3 G^* (i\omega)$ . The results are also plotted in figure 2. The comparison is excellent throughout most of the frequency regime.

The data was further analyzed to determine the sum interval contributions in equations (9), (10) and (12). These results are given in table II for  $\omega = 100$  cps. Four different frequencies were run, .01, 1, 100, and 1000 cps, to determine the interval sums up to  $t = \frac{10 (2\pi) + \pi/2}{\omega}$  in equation (9) and  $t = 10(\frac{2\pi}{\omega})$  in equation (10). In all cases, the sum contributions added up to results accurate to four significant figures as compared with the final computed answers for  $E' (\omega)$  and  $E'' (\omega)$  using the full time regime. For  $\omega = 1$  cps,  $10(\frac{2\pi}{\omega}) = 10$  seconds. Thus an accurate computation for  $E^* (i\omega)$  could be obtained from stress relaxation tests run for only 10 seconds.

The minimum time at which the first data point must be known in order to compute  $E' (\omega)$  and  $E'' (\omega)$  accurately to four significant figures for  $\omega = 1$  cps is  $1 \times 10^{-3}$  sec. for  $E' (\omega)$  and  $1 \times 10^{-5}$  sec. for  $E'' (\omega)$ .

The time range over which the stress relaxation modulus for polyisobutylene must be known to compute  $E^* (i\omega)$  accurate to four significant figures for a given frequency is approximately 5 decades for  $E' (\omega)$  and 10 decades for  $E'' (\omega)$ . This is true for the frequency range from .01 to 1000 cps, which is presented graphically in figure 3. These limits apply only to polyisobutylene but those for solid propellant should be similar.

---

<sup>4,5</sup>

See List of References

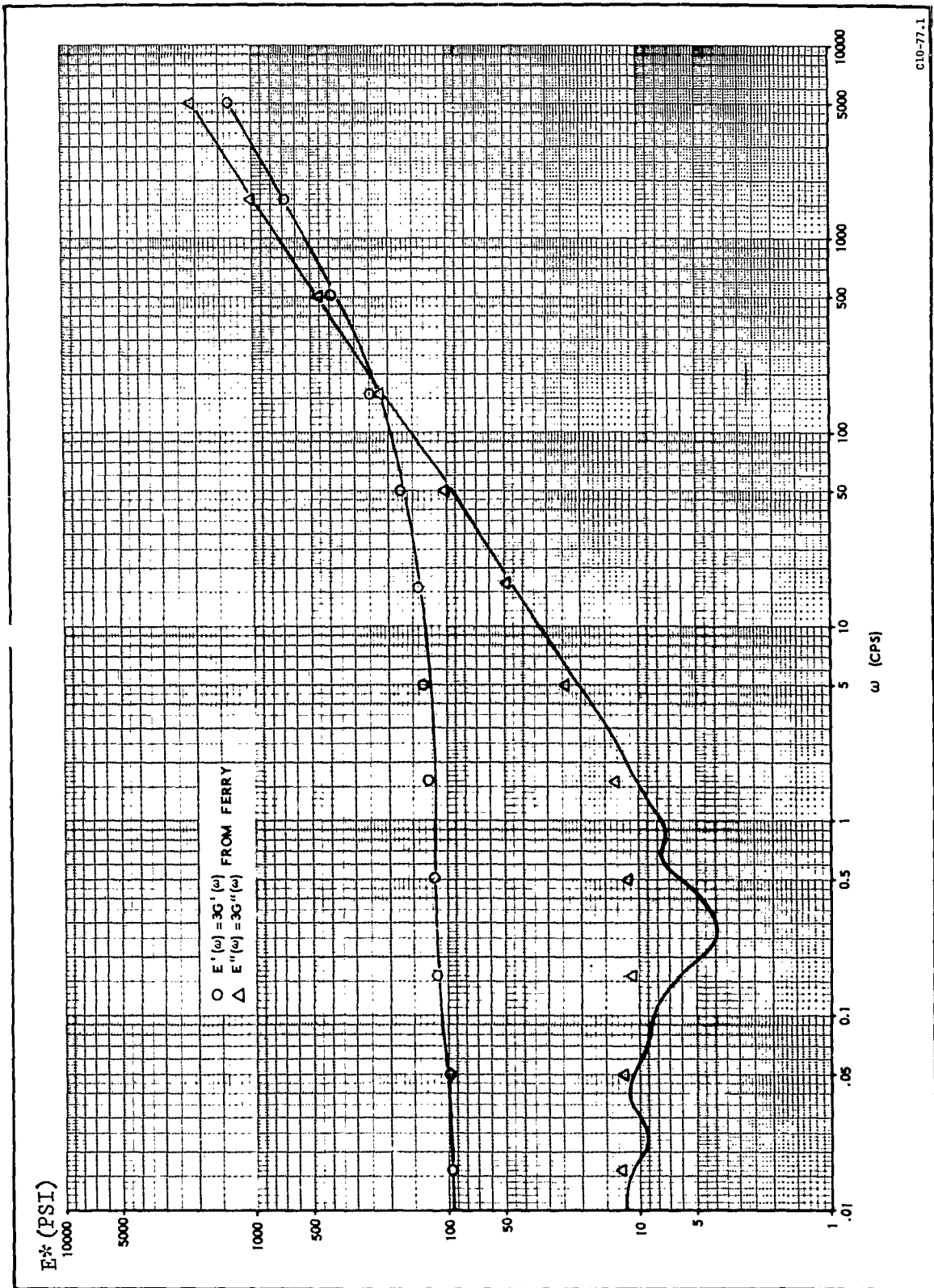
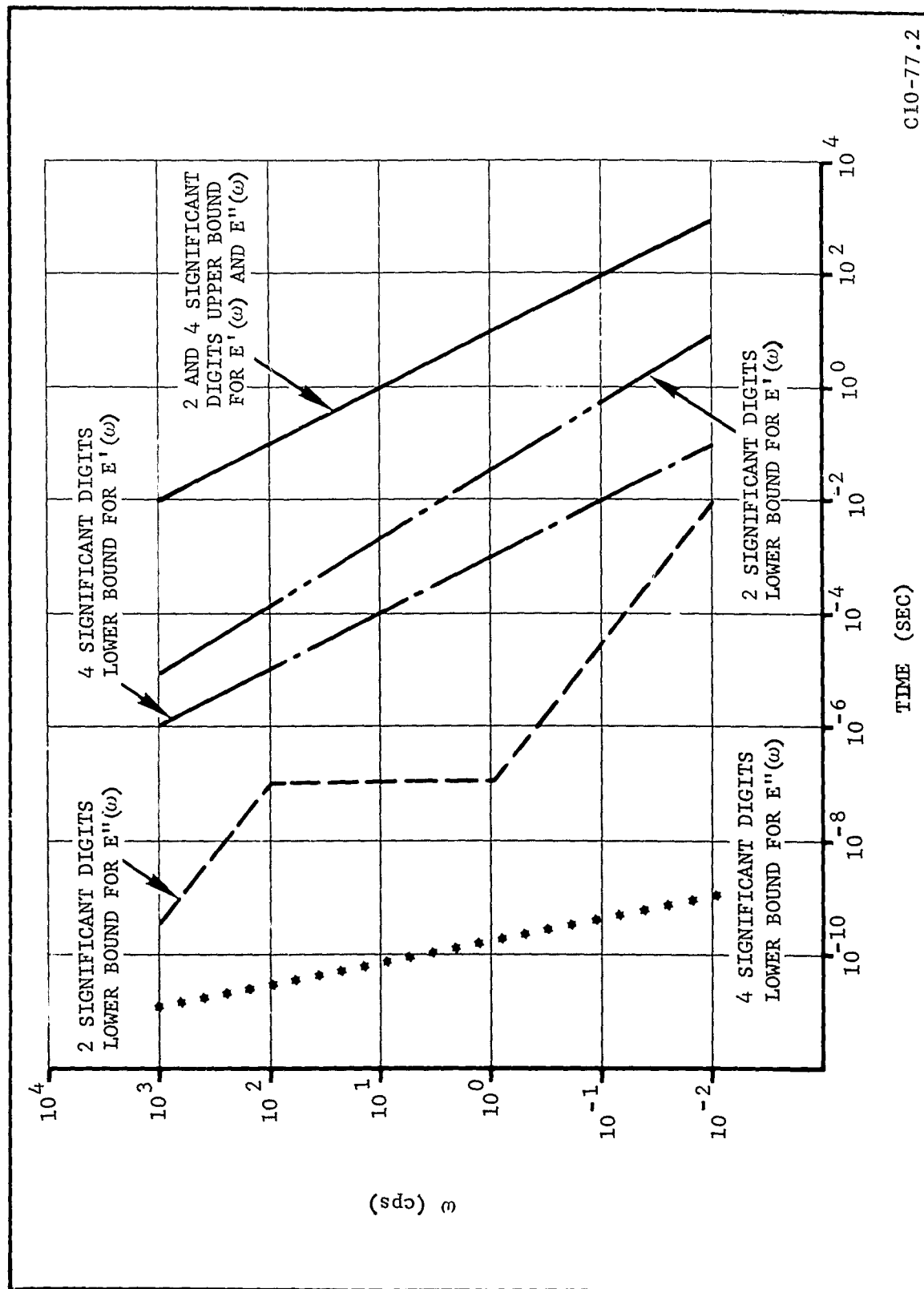


Figure 2.  $E^*(i\omega)$  for Polyisobutylene



C10-77.2

Figure 3. Time Regime in Which  $E(t)$  Must Be Known to Compute  $E^*(i\omega)$  for Polyisobutylene

TABLE I

## STRESS RELAXATION DATA FOR POLYSIOBUTYLENE

| $\log_{10} t$<br>(HRS) | $\log_{10} E(t)$<br>(PSI) | $\log_{10} t$<br>(HRS) | $\log_{10} E(t)$<br>(PSI) |
|------------------------|---------------------------|------------------------|---------------------------|
| <del>-.45000E+02</del> | <del>.56500E+01</del>     | <del>-.48000E+01</del> | <del>.20660E+01</del>     |
| <del>-.14400E+02</del> | <del>.56460E+01</del>     | <del>-.46000E+01</del> | <del>.20610E+01</del>     |
| <del>-.14200E+02</del> | <del>.56410E+01</del>     | <del>-.44000E+01</del> | <del>.20560E+01</del>     |
| <del>-.14000E+02</del> | <del>.56260E+01</del>     | <del>-.42000E+01</del> | <del>.20510E+01</del>     |
| <del>-.13800E+02</del> | <del>.56160E+01</del>     | <del>-.40000E+01</del> | <del>.20460E+01</del>     |
| <del>-.13600E+02</del> | <del>.56060E+01</del>     | <del>-.38000E+01</del> | <del>.20410E+01</del>     |
| <del>-.13400E+02</del> | <del>.55910E+01</del>     | <del>-.36000E+01</del> | <del>.20360E+01</del>     |
| <del>-.13200E+02</del> | <del>.55710E+01</del>     | <del>-.34000E+01</del> | <del>.20310E+01</del>     |
| <del>-.13000E+02</del> | <del>.55510E+01</del>     | <del>-.32000E+01</del> | <del>.20160E+01</del>     |
| <del>-.12800E+02</del> | <del>.55310E+01</del>     | <del>-.30000E+01</del> | <del>.20010E+01</del>     |
| <del>-.12600E+02</del> | <del>.55010E+01</del>     | <del>-.28000E+01</del> | <del>.19860E+01</del>     |
| <del>-.12400E+02</del> | <del>.54610E+01</del>     | <del>-.26000E+01</del> | <del>.19710E+01</del>     |
| <del>-.12200E+02</del> | <del>.54160E+01</del>     | <del>-.24000E+01</del> | <del>.19560E+01</del>     |
| <del>-.12000E+02</del> | <del>.53660E+01</del>     | <del>-.22000E+01</del> | <del>.19410E+01</del>     |
| <del>-.11800E+02</del> | <del>.53110E+01</del>     | <del>-.20000E+01</del> | <del>.19160E+01</del>     |
| <del>-.11600E+02</del> | <del>.52360E+01</del>     | <del>-.18000E+01</del> | <del>.18960E+01</del>     |
| <del>-.11400E+02</del> | <del>.51410E+01</del>     | <del>-.16000E+01</del> | <del>.18710E+01</del>     |
| <del>-.11200E+02</del> | <del>.50460E+01</del>     | <del>-.14000E+01</del> | <del>.18460E+01</del>     |
| <del>-.11000E+02</del> | <del>.49360E+01</del>     | <del>-.12000E+01</del> | <del>.18160E+01</del>     |
| <del>-.10800E+02</del> | <del>.48160E+01</del>     | <del>-.10000E+01</del> | <del>.17860E+01</del>     |
| <del>-.10600E+02</del> | <del>.46860E+01</del>     | <del>-.80000E+00</del> | <del>.17510E+01</del>     |
| <del>-.10400E+02</del> | <del>.45560E+01</del>     | <del>-.60000E+00</del> | <del>.17110E+01</del>     |
| <del>-.10200E+02</del> | <del>.44210E+01</del>     | <del>-.40000E+00</del> | <del>.16660E+01</del>     |
| <del>-.10000E+02</del> | <del>.42860E+01</del>     | <del>-.20000E+00</del> | <del>.16160E+01</del>     |
| <del>-.98000E+01</del> | <del>.41510E+01</del>     | <del>.00000E+00</del>  | <del>.15560E+01</del>     |
| <del>-.96000E+01</del> | <del>.40260E+01</del>     | <del>.20000E+00</del>  | <del>.14910E+01</del>     |
| <del>-.94000E+01</del> | <del>.38960E+01</del>     | <del>.40000E+00</del>  | <del>.14210E+01</del>     |
| <del>-.92000E+01</del> | <del>.37660E+01</del>     | <del>.60000E+00</del>  | <del>.13360E+01</del>     |
| <del>-.90000E+01</del> | <del>.36310E+01</del>     | <del>.80000E+00</del>  | <del>.12410E+01</del>     |
| <del>-.88000E+01</del> | <del>.34910E+01</del>     | <del>.10000E+01</del>  | <del>.11310E+01</del>     |
| <del>-.86000E+01</del> | <del>.33560E+01</del>     | <del>.12000E+01</del>  | <del>.10110E+01</del>     |
| <del>-.84000E+01</del> | <del>.32160E+01</del>     | <del>-.14000E+01</del> | <del>.86100E+00</del>     |
| <del>-.82000E+01</del> | <del>.30860E+01</del>     | <del>.16000E+01</del>  | <del>.70100E+00</del>     |
| <del>-.80000E+01</del> | <del>.29660E+01</del>     | <del>.18000E+01</del>  | <del>.56100E+00</del>     |
| <del>-.78000E+01</del> | <del>.28360E+01</del>     | <del>.20000E+01</del>  | <del>.34100E+00</del>     |
| <del>-.76000E+01</del> | <del>.27460E+01</del>     | <del>.22000E+01</del>  | <del>.61000E+00</del>     |
| <del>-.74000E+01</del> | <del>.26460E+01</del>     | <del>.24000E+01</del>  | <del>.33900E+00</del>     |
| <del>-.72000E+01</del> | <del>.25460E+01</del>     | <del>.26000E+01</del>  | <del>.83900E+00</del>     |
| <del>-.70000E+01</del> | <del>.24560E+01</del>     |                        |                           |
| <del>-.68000E+01</del> | <del>.23710E+01</del>     |                        |                           |
| <del>-.66000E+01</del> | <del>.23010E+01</del>     |                        |                           |
| <del>-.64000E+01</del> | <del>.22460E+01</del>     |                        |                           |
| <del>-.62000E+01</del> | <del>.22010E+01</del>     |                        |                           |
| <del>-.60000E+01</del> | <del>.21660E+01</del>     |                        |                           |
| <del>-.58000E+01</del> | <del>.21410E+01</del>     |                        |                           |
| <del>-.56000E+01</del> | <del>.21210E+01</del>     |                        |                           |
| <del>-.54000E+01</del> | <del>.21010E+01</del>     |                        |                           |
| <del>-.52000E+01</del> | <del>.20860E+01</del>     |                        |                           |
| <del>-.50000E+01</del> | <del>.20760E+01</del>     |                        |                           |

## NOTE:

PRINTOUTS MAY BE PUT INTO A MORE READABLE FORM. FOR EXAMPLE THE PRINTOUT ~~-.45000E + 02~~ BECOMES -- 45.0, BECAUSE THE E + 02 OF THE PRINTOUT DESIGNATES THE NUMBER AND DIRECTION OF THE DECIMAL POINT. NOTE THE FOLLOWING EXAMPLES:

$$\begin{aligned}
 & \pm 0.10600 \text{ E } + 02 = \pm 10.6 \\
 & \pm 0.82000 \text{ E } + 01 = \pm 8.20 \\
 & \pm 0.40000 \text{ E } - 00 = \pm 0.40 \\
 & \pm 0.00000 \text{ E } - 99 = 0 \\
 & \pm 0.12000 \text{ E } - 01 = \pm 0.012 \\
 & \pm 0.23000 \text{ E } - 02 = \pm 0.0023
 \end{aligned}$$

C10-92.1

TABLE II

INTERVAL CONTRIBUTIONS FOR  $E^*(i\omega)$ 

| $T_n$      | $E'_{n+1}$ | $E''_{n+1}$ | $E'_{n+1} - E'_n$ | $E''_{n+1} - E''_n$ |
|------------|------------|-------------|-------------------|---------------------|
| .99999E-45 | .44877E-01 | .40039E-02  | .18029E-10        | .40039E-02          |
| .39810E-14 | .44877E-01 | .63216E-02  | .26963E-10        | .23177E-02          |
| .63095E-14 | .44877E-01 | .99115E-02  | .66131E-10        | .35898E-02          |
| .10000E-13 | .44877E-01 | .15439E-01  | .16146E-09        | .55280E-02          |
| .15848E-13 | .44877E-01 | .24001E-01  | .39639E-09        | .85619E-02          |
| .25118E-13 | .44877E-01 | .37186E-01  | .96695E-09        | .13185E-01          |
| .39810E-13 | .44877E-01 | .57258E-01  | .23320E-08        | .20072E-01          |
| .63095E-13 | .44877E-01 | .87639E-01  | .55941E-08        | .30380E-01          |
| .10000E-12 | .44877E-01 | .13362E-00  | .13419E-07        | .45983E-01          |
| .15848E-12 | .44877E-01 | .20243E-00  | .31798E-07        | .68809E-01          |
| .25118E-12 | .44877E-01 | .30305E-00  | .73637E-07        | .10062E-00          |
| .39810E-12 | .44877E-01 | .44768E-00  | .16766E-06        | .14462E-00          |
| .63095E-12 | .44877E-01 | .65318E-00  | .37740E-06        | .20549E-00          |
| .10000E-11 | .44878E-01 | .94181E-00  | .83978E-06        | .28863E-00          |
| .15848E-11 | .44880E-01 | .13359E+01  | .18140E-05        | .39409E-00          |
| .25118E-11 | .44884E-01 | .18498E+01  | .37430E-05        | .51395E-00          |
| .39810E-11 | .44891E-01 | .25043E+01  | .75548E-05        | .65452E-00          |
| .63095E-11 | .44906E-01 | .33242E+01  | .14977E-04        | .81982E-00          |
| .10000E-10 | .44935E-01 | .43217E+01  | .28859E-04        | .99756E-00          |
| .15848E-10 | .44989E-01 | .55080E+01  | .54344E-04        | .11862E+01          |
| .25118E-10 | .45091E-01 | .69017E+01  | .10119E-03        | .13937E+01          |
| .39810E-10 | .45278E-01 | .85303E+01  | .18732E-03        | .16285E+01          |
| .63095E-10 | .45623E-01 | .10421E+02  | .34481E-03        | .18914E+01          |
| .10000E-09 | .46258E-01 | .12618E+02  | .63472E-03        | .21968E+01          |
| .15848E-09 | .47440E-01 | .15196E+02  | .11822E-02        | .25796E+01          |
| .25118E-09 | .49654E-01 | .18247E+02  | .22138E-02        | .30491E+01          |
| .39810E-09 | .53776E-01 | .21829E+02  | .41224E-02        | .35824E+01          |
| .63095E-09 | .61407E-01 | .26015E+02  | .76311E-02        | .41860E+01          |
| .10000E-08 | .75372E-01 | .30851E+02  | .13964E-01        | .48353E+01          |
| .15848E-08 | .10093E-00 | .36433E+02  | .25561E-01        | .55820E+01          |
| .25118E-08 | .14770E-00 | .42881E+02  | .46775E-01        | .64478E+01          |
| .39810E-08 | .23383E-00 | .50365E+02  | .86124E-01        | .74840E+01          |
| .63095E-08 | .39610E-00 | .59254E+02  | .16227E-00        | .88889E+01          |
| .10000E-07 | .70897E-00 | .70056E+02  | .31287E-00        | .10802E+02          |
| .15848E-07 | .13188E+01 | .83337E+02  | .60990E-00        | .13280E+02          |
| .25118E-07 | .25216E+01 | .97830E+02  | .12027E+01        | .16493E+02          |
| .39810E-07 | .49181E+01 | .12050E+03  | .23964E+01        | .20678E+02          |
| .63095E-07 | .97399E+01 | .14656E+03  | .48218E+01        | .26051E+02          |
| .10000E-06 | .19555E+02 | .17943E+03  | .98158E+01        | .32871E+02          |
| .15848E-06 | .39736E+02 | .22011E+03  | .20180E+02        | .40678E+02          |
| .25118E-06 | .81201E+02 | .26636E+03  | .41465E+02        | .46257E+02          |
| .39810E-06 | .16104E+03 | .30121E+03  | .79845E+02        | .34847E+02          |
| .63095E-06 | .28022E+03 | .26844E+03  | .11928E+03        | .32768E+02          |
| .10000E-05 | .31929E+03 | .97923E+02  | .38971E+02        | .17052E+03          |
| .15848E-05 | .85903E+02 | .77392E+02  | .23339E+03        | .20530E+02          |
| .25118E-05 | .30820E+03 | .20661E+03  | .22230E+03        | .12922E+03          |
| .39810E-05 | .20894E+03 | .27530E+03  | .99262E+02        | .68688E+02          |
| .63095E-05 | .28942E+03 | .84783E+02  | .80486E+02        | .19052E+03          |
| .10000E-04 | .22519E+03 | .42794E+02  | .64235E+02        | .41988E+02          |
| .15848E-04 | .82191E+02 | .18515E+03  | .14300E+03        | .14236E+03          |

C10-92.2

TABLE II

INTERVAL CONTRIBUTIONS FOR  $E^*(i, \omega)$  (Cont)

| $T_n$      | $E'_{n+1}$ | $E''_{n+1}$ | $E'_{n+1} - E'_n$ | $E''_{n+1} - E''_n$ |
|------------|------------|-------------|-------------------|---------------------|
| .25118E-04 | .19306E+03 | .15458E+03  | .11087E+03        | .30570E+02          |
| .39810E-04 | .19306E+03 | .15458E+03  | .28476E-03        | .28476E-03          |
| .63095E-04 | .19306E+03 | .15458E+03  | .17761E-03        | .17761E-03          |
| .10000E-03 | .19306E+03 | .15458E+03  | .11078E-03        | .11078E-03          |
| .15848E-03 | .19306E+03 | .15459E+03  | .69100E-04        | .69100E-04          |
| .25118E-03 | .19306E+03 | .15459E+03  | .43102E-04        | .43102E-04          |
| .39810E-03 | .19306E+03 | .15459E+03  | .23919E-03        | .23919E-03          |
| .63095E-03 | .19306E+03 | .15459E+03  | .14580E-03        | .14580E-03          |
| .10000E-02 | .19306E+03 | .15459E+03  | .88874E-04        | .88874E-04          |
| .15848E-02 | .19306E+03 | .15459E+03  | .54172E-04        | .54172E-04          |
| .25118E-02 | .19306E+03 | .15459E+03  | .33020E-04        | .33020E-04          |
| .39810E-02 | .19306E+03 | .15459E+03  | .20127E-04        | .20127E-04          |
| .63095E-02 | .19306E+03 | .15459E+03  | .33693E-04        | .33693E-04          |
| .10000E-01 | .19306E+03 | .15459E+03  | .12918E-04        | .12918E-04          |
| .15848E-01 | .19306E+03 | .15459E+03  | .12093E-04        | .12093E-04          |
| .25118E-01 | .19306E+03 | .15459E+03  | .72041E-05        | .72041E-05          |
| .39810E-01 | .19306E+03 | .15459E+03  | .61445E-05        | .61445E-05          |
| .63095E-01 | .19306E+03 | .15459E+03  | .36183E-05        | .36183E-05          |
| .10000E-00 | .19306E+03 | .15459E+03  | .28837E-05        | .28837E-05          |
| .15848E-00 | .19306E+03 | .15459E+03  | .21802E-05        | .21802E-05          |
| .25118E-00 | .19306E+03 | .15459E+03  | .15790E-05        | .15790E-05          |
| .39810E-00 | .19306E+03 | .15459E+03  | .11027E-05        | .11027E-05          |
| .63095E-00 | .19306E+03 | .15459E+03  | .88313E-06        | .88313E-06          |
| .10000E+01 | .19306E+03 | .15459E+03  | .56648E-06        | .56648E-06          |
| .15848E+01 | .19306E+03 | .15459E+03  | .35499E-06        | .35499E-06          |
| .25118E+01 | .19306E+03 | .15459E+03  | .27651E-06        | .27651E-06          |
| .39810E+01 | .19306E+03 | .15459E+03  | .17729E-06        | .17729E-06          |
| .63095E+01 | .19306E+03 | .15459E+03  | .11860E-06        | .11860E-06          |
| .99999E+01 | .19306E+03 | .15459E+03  | .68435E-07        | .68435E-07          |
| .15848E+02 | .19306E+03 | .15459E+03  | .49600E-07        | .49600E-07          |
| .25118E+02 | .19306E+03 | .15459E+03  | .24993E-07        | .24993E-07          |
| .39810E+02 | .19306E+03 | .15459E+03  | .16395E-07        | .16395E-07          |
| .63095E+02 | .19306E+03 | .15459E+03  | .43859E-08        | .43859E-08          |
| .99999E+02 | .19306E+03 | .15459E+03  | .52283E-08        | .52283E-08          |
| .15848E+03 | .19306E+03 | .15459E+03  | .32516E-08        | .32516E-08          |
| .25118E+03 | .19306E+03 | .15459E+03  | .13375E-08        | .13375E-08          |

## NOTE:

PRINTOUTS MAY BE PUT INTO A MORE READABLE FORM.  
 FOR EXAMPLE THE PRINTOUT - 0.045000 E + 02  
 BECAME - 45.0, BECAUSE THE E + 02 OF THE  
 PRINTOUT DESIGNATES THE NUMBER AND DIRECTION  
 OF THE DECIMAL POINT. NOTE THE FOLLOWING EXAMPLES:

$\pm 0.10600 \text{ E} + 02 = \pm 10.6$   
 $\pm 0.82000 \text{ E} + 01 = \pm 8.20$   
 $\pm 0.40000 \text{ E} - 00 = \pm 0.40$   
 $\pm 0.00000 \text{ E} - 99 = 0$   
 $\pm 0.12000 \text{ E} - 01 = \pm 0.012$   
 $\pm 0.23000 \text{ E} - 02 = \pm 0.0023$

C10-92.3

#### LIST OF REFERENCES

1. Bergen, J. T., "Viscoelasticity - Phenomenological Aspects," Academic Press Inc., New York City (1960) P. 36
2. Ferry, J. D., "Viscoelastic Properties of Polymers," John Wiley and Sons, Inc., New York City (1961) P. 54
3. Ferry, J. D., "Viscoelastic Properties of Polymers," John Wiley and Sons, Inc., New York City (1961) P. 44
4. Tobolsky, A. V., "Properties and Structure of Polymers," John Wiley and Sons, Inc., New York City (1960) P. 152
5. Ferry, J. D., "Viscoelastic Properties of Polymers," John Wiley and Sons, Inc., New York City (1961) P. 457

# LIST OF SYMBOLS

|                |  |
|----------------|--|
| $\omega$       | ... circular frequency   |
| $t$            | ... time   |
| $E(t)$         | ... stress relaxation modulus  |
| $E^*(i\omega)$ | ... dynamic modulus  |
| $E'(\omega)$   | ... real part of dynamic modulus                                       |
| $E''(\omega)$  | ... imaginary part of dynamic modulus                                  |
| $E_c$          | ... constant value approach by stress relaxation modulus for large $t$ |
| $\psi(t)$      | ... $E(t) - E_c$   |
| $m$            | ... see equation (5)   |
| $a_n$          | ... see equation (10)  |
| $b_n$          | ... see equation (10)  |
| $t_n$          | ... see equation (10)  |
| $p$            | ... see equation (11)  |
| $q$            | ... see equation (11)  |
| $n_1$          | ... see equation (7)   |
| $n_2$          | ... see equation (8)   |



# APPENDIX A

## 1620 COMPUTER PROGRAM

### 1. OPERATING INSTRUCTIONS AND INPUT DATA FORMAT

Card. No.

- |       |                   |  |
|-------|-------------------|--|
| 1     | Title card        | - Any Hollerith Information in first 45 columns of card  |
| 2     | Parameter card    | - NT, NW, SCL, <span style="float: right;">FORMAT</span><br>(2I5,E 14.8)<br>NT = number of time increments in stress relaxation table.<br>NW = multiple of $2\frac{\pi}{\omega}$ at which the program will begin using equation (12).<br><br>SCL = scale factor to multiply input frequency by in order to make the frequency and stress relaxation data in compatible units; SCL is assumed to be $2\pi$ if columns 11-25 are left blank. |
| 3     | Stress Relaxation | - $\log_{10} (t_1), \log_{10} (E(t_1))$  |
| 4     | Table             | $\log_{10} (t_2), \log_{10} (E(t_2))$  |
| 5     |                   | $\log_{10} (t_3), \log_{10} (E(t_3))$  |
| .     |                   | .  |
| .     |                   | .  |
| .     |                   | .  |
| .     |                   | .  |
| M+2   |                   | - $\log_{10} (t_M), \log_{10} (E(t_M))$  |
| M+3   | Frequencies       | $\omega_1$   |
| M+4   |                   | $\omega_2$   |
| M+5   |                   | $\omega_3$   |
| .     |                   | .  |
| .     |                   | .  |
| .     |                   | .  |
| .     |                   | .  |
| M+N+2 |                   | $\omega_n$   |
| M+N+3 |                   | blank card (if new stress relaxation table is to be read in)   |
| M+N+4 | Title card        |  |
| M+N+5 | Parameter card    |  |
| .     | etc.              |  |
| .     | .                 |  |
| .     | .                 |  |
- Format for Frequencies and Stress Relaxation Table (2E10.5)

## 2. PROGRAM LISTING

```

E STAR PROGRAM
DIMENSION TN(90), EN(90), AN(90), BN(90)
TPI = 6.2831853
TEN = 10.
1  READ 10
   PRINT 10
   PUNCH 10
10  FORMAT (45H
      READ 20, NT, NW, SCL
      PUNCH 20, NT, NW, SCL
20  FORMAT (2I5,E14.8)
      TRA = NW
      TRA = TRA*TPI
      IF (SCL) 28, 24, 28
24  SCL = TPI
28  DO 30 I = 1, NT
      READ 40, TN (I), EN (I)
30  PUNCH 50, TN (I), EN (I)
40  FORMAT (6E10.5)
50  FORMAT (6E11.5)
      TN (1) = TEN**TN (1)
      T1 = TN (1)
      EN (1) = TEN**EN (1)
      E1 = EN (1)
      DO 60 I = 2, NT
      TN (I) = TEN**TN(I)
      T2 = TN (I)
      EN (I) = TEN**EN (I)
      E2 = EN (I)
      BN (I) = LOGF (E1/E2)/(T2-T1)
      AN (I) = E1* EXPF (BN(I)*T1)
      T1 = T2
60  E1 = E2
70  READ 40, WC
      IF (WC) 80, 1, 80
80  PUNCH 50, WC
100 W = WC*SCL
110 EP = 0.
      EPP = 0.
      KSW = 0
      TRAW = TRA/W
      W2 = W*W
      T1 = TN (1)
      DO 170 I = 2, NT
      T2 = TN (I)
      IF (T2-TRAW) 120, 120, 130

```

PROGRAM LISTING (Cont)

```

120  WT = W*T2
      CW = COSF (WT)
      CX = CW
      SW = SINP (WT)
      SX = SW
125  WT = W*T1
      CWT = COSF (WT)
      SWT = SINP (WT)
      TE = AN (I)*W/(BN(I)*BN(I) + W2)
      TT = EXPF (BN(I)*T1)
      FN = (BN(I)*SWT+W*CWT)/TT
      FNP = (BN(I)*CWT-W*SWT)/TT
      TT = EXPF (BN(I)*T2)
      FN = TE*(FN-(BN(I)*SW+W*CW)/TT)
      FNP = TE*(FNP-(BN(I)*CX-W*SX)/TT)
      GO TO 160
130  IF (KSW) 150, 140, 150
140  CW = 0.
      SW = 1.
      CX = 1.
      SX = 0.
      KSW = 1
      GO TO 125
150  BNW = BN(I)/W
      FN = BNW*(EN(I-1)-EN(I))/(1.+BNW**2)
      FNP = FN
160  EP = EP+FN
      EPP = EPP+FNP
170  T1 = T2
      PRINT 180, WC, EP, FN
      PUNCH 180, WC, EP, FN
180  FORMAT (5H EP(F8.2,2H) = E11.5, 12H LAST TERM = E11.5
      PRINT 190, WC, EPP, FNP
      PUNCH 190, WC, EPP, FNP
190  FORMAT (5H EPP (F8.2,2H) = E11.5, 12H LAST TERM = E11.5
      GO TO 70
      END

```

DISTRIBUTION LIST  
CONVERSION OF STRESS RELAXATION MODULUS TO  
DYNAMIC MODULUS FOR A VISCOELASTIC MATERIAL  
MTO-270-2

| <u>Copy No.</u>  | <u>Recipient</u>  | <u>Mail Stop</u> |
|------------------|---|------------------|
| 1<br>thru<br>10  | Headquarters<br>Space System Division<br>United States Air Force<br>Air Force Unit Post Office<br>Los Angeles 45, California<br>Attention: SSSC   |                  |
| 11<br>thru<br>15 | Space Technology Laboratories<br>P. O. Blx 1310<br>Norton Air Force Base<br>San Bernardino, California<br>Attention: Mr. J. R. McGibbeny, Manager<br>Stage III MINUTEMAN Propulsion<br>For: Mr. R. W. Sears |                  |
| 16               | Space Technology Laboratories<br>One Space Park<br>Redondo Beach, California<br>Attention: Mr. J. R. McGibbeny, Manager<br>Stage III MINUTEMAN Propulsion<br>For: Mr. R. L. Greengard                       |                  |
| 17               | HP/CMO<br>Bacchus Works<br>Magna, Utah<br>Attention: Lt. Col. G. E. Scott, AFOIC  |                  |
| 18               | Chief, Philadelphia Contract Management District<br>United States Air Force<br>1411 Walnut Street<br>Philadelphia 2, Pennsylvania<br>Attention: Mr. S. E. Patti   |                  |
| 19               | The Boeing Company<br>Aero-Space Division<br>P. O. Box 3985<br>Seattle 24, Washington<br>Attention: Mr. T. A. Wilson  |                  |

Distribution List (Cont)

Copy No.

Mail Stop

|    |   |
|----|---|
| 20 | Mr. W. E. Howell, Director of Contract Adm.<br>in turn to: Mr. M. C. Burgy<br>Dr. L. G. Bonner<br>Chemical Propulsion Division<br>Hercules Powder Company<br>Hercules Tower<br>910 Market Street<br>Wilmington 99, Delaware |
| 21 | Mr. J. E. Greer, Director of Air Force Programs<br>Chemical Propulsion Division<br>Hercules Powder Company<br>Hercules Tower<br>910 Market Street<br>Wilmington 99, Delaware  |
| 22 | Mr. J. L. Shrout<br>Los Angeles Sales Office<br>3460 Wilshire Blvd.<br>Los Angeles 5, California  |
| 23 | Mr. J. C. Foster, Manager<br>Allegany Ballistic Laboratories<br>Cumberland, Maryland  |
| 24 | Mr. N. L. McLeod, Manager<br>Hercules Powder Company<br>Kenvil Works<br>Kenvil, New Jersey  |
| 25 | Mr. B. H. Sleight, Jr., Manager<br>Hercules Powder Company<br>Radford Ordnance Plant<br>Radford, Virginia   |
| 26 | Dr. Dwight F. Gunder<br>463 West 5th Street<br>Loveland, Colorado   |
| 27 | Mr. L. B. Johnston, Manager<br>Hercules Powder Company<br>Rocky Hill Plant<br>Rocky Hill, New Jersey  |

Distribution List (Cont)

| <u>Copy No.</u>   | <u>Recipient</u>             | <u>Mail Stop</u> |
|-------------------|------------------------------|------------------|
| 28                |                              |                  |
| thru              | J. M. Anderson               | 117              |
| 42                |                              |                  |
| 43                | W. M. Beebe                  | 117-T-43         |
| 44                | D. E. Boynton                | 100-G-2-1        |
| 45                | R. N. Chappell               | 117-T-48         |
| 46                | H. L. Holt                   | 117-T-43         |
| 47                | R. E. Keeffe                 | 506              |
| 48                | E. B. Magrab                 | 117-T-43         |
| 49                | J. H. Main                   | 505              |
| 50                | L. F. Myers                  | 117-T-48         |
| 51                | M. D. Nay                    | 117-T-48         |
| 52                | A. H. Peterson               | 112              |
| 53                | D. E. Richardson             | 117-T-48         |
| 54                | J. N. Sherman                | 505              |
| 55                | S. R. Swanson                | 117-T-48         |
| 56                | T. L. Vose                   | 117-T-43         |
| 57                |                              |                  |
| and               | Publications Library         | 700              |
| 58                |                              |                  |
| 59 (reproducible) | Publications Library         | 700              |
| 60                |                              |                  |
| and               | Library/Central File         | 509-C            |
| 61                |                              |                  |
|                   | J. L. Morse (letter only)    | 502              |
|                   | A. H. Nielson (letter only)  | 81-16-2          |
|                   | J. L. Andrew (letter only)   | 505              |
|                   | M. Cohen (letter only)       |                  |
| 62                | R. L. Griffis (J. R. Larson) | 700              |